Modulation bandwidth of metal-clad semiconductor nanolasers with cavity-enhanced spontaneous emission

K.A. Shore

Analytical approximations for the direct-current modulation bandwidth are obtained which highlight the deleterious effect of enhanced modal refractive index on the resonance frequency of metal-clad nanolasers.

The concomitant utilisation of Purcell cavity-enhanced spontaneous emission is shown, in principle, to compromise achievable modulation bandwidths further. Using simplified forms for the modulation bandwidth, estimates are made which suggest significant impact on the achievable dynamical response of such lasers. The presented analysis may assist in the design of semiconductor nanolasers.

Introduction: The development of semiconductor nanolasers has received increasing attention in recent years. The main driver for such activity is the need for miniaturised lasers compatible with general applications of nanophotonics. A highly accessible introduction to the principles of nanolasers is available in a recent tutorial paper by Ning [1]. A particular focus of the paper is metal-semiconductor plasmonic lasers where rather subtle waveguiding effects arise. In particular, careful attention is required in the definition of the modal gain and confinement factor in such systems. In previous work, Maslov and Ning carefully examined the modal gain in a semiconductor nanowire laser [2]. Attention to the confinement factor in nanolasers has been paid in extensive work by Chang and Chuang [3]. Chang, Lin and Chuang [4] have developed a comprehensive approach to the design and performance evaluation of plasmonic Fabry-Perot nanolasers. The latter work included a rate equation formulation for the device dynamics. In addition to the role played by plasmonic effects in determining the performance of nanolasers, the very utilisation of nanocavities opens the opportunity to exploit the Purcell effect wherein enhanced spontaneous emission can be used to achieve laser threshold reduction and threshold-less lasing [5]. Limits on practical attainment of such behaviour in semiconductor micro-cavity lasers have been previously noted [6]. In recent work, the impact of Purcell enhanced spontaneous emission on the modulation performance of nanoLEDs [7] and nanolasers [8] has been examined. In particular Suhr et al. [8] have performed a detailed analysis of the Purcell effect for quantum well nanoLEDs and nanolasers and have shown that the Purcell enhancement saturates, thus limiting the achievable spontaneous recombination rate and hence imposing a limitation of modulation bandwidths to the of order 10 GHz.

In this Letter, the aim is to perform a simplified synthesis of the combined impact on modulation bandwidth of the enhanced confinement factor and enhanced spontaneous emission in metal-clad nanolasers. The focus of the work is to extract approximate expressions which illustrate the detrimental consequences for the modulation behaviour of such lasers of combining these effects. The expressions obtained will give useful guidelines for metal-clad laser design, albeit with need for more detailed analysis of waveguide modes and material gain to obtain a full assessment of device dynamics.

Modulation bandwidth: The analysis proceeds from rate equations for the carrier density, N, and photon density, S, in the form:

\[
\frac{dN}{dt} = J - FN\beta / \tau_c - (1 - \beta N) / \tau_p - v_o G_0 (N - N_0) S
\]

(1)

where J is the normalised injection current; F is the Purcell spontaneous emission enhancement factor; \(\beta\) is the spontaneous emission coupling factor; \(\tau_c\) is the carrier lifetime; \(v_o = c/n_o\) with \(n_o\) the background refractive index of the active region; \(G_0\) is the differential gain coefficient; \(N_0\) is the transparency carrier density.

\[
\frac{dS}{dt} = C_0 BFN / \tau_c + C_0 v_o G_0 (N - N_0) S - N / \tau_p
\]

(2)

where \(C_0\) is the conventional optical power confinement factor; \(\tau_p\) is the photon lifetime; here

\[
\tau_p^{-1} = \frac{1}{n_2} \ln (1/R_{S} |R| - 1) [L]
\]

(3)

where \(n_2 = c / n_2\) with \(n_2\) the group refractive index.

In general the direct-current modulation bandwidth can be obtained in the form [8, 9]:

\[
f_{3dB} = 1/2 \pi \sqrt{|(\omega_0 - \gamma_0)/2|}
\]

(4)

where \(\omega_0 = \omega_0 - \gamma_0/2\) with \(\omega_0\) and \(\gamma_0\) being the resonance frequency and damping factor, respectively.

In the above rate equations, to make the analysis more transparent, albeit a little less generally applicable, omission has been made of a number of terms included in the comprehensive analysis reported in [9]. Specifically non-radiative recombination and nonlinear optical gain have been suppressed. Then we have:

\[
\omega_0 = 1 / \tau_0 (G_0 S_0 + F \beta / \tau_c) + C_0 (1 - \beta) F N_0 / (S_0 \tau_p^2)
\]

(5)

\[
\gamma_0 = G_0 S_0 + (1 - \beta \tau_c + F \beta / \tau_c (1 + C_0 N_0 / S_0))
\]

(6)

where \(S_0\) is the steady state photon density.

The gross trend of the impact of metal-cladding and Purcell effect enhanced spontaneous emission can be obtained from a consideration of \(\omega_0\) which encapsulates the fundamental damped harmonic oscillation dynamics of the semiconductor laser. Thus focusing on \(\omega_0\) we have

\[
f_{3dB} = 1/2 \pi \sqrt{|(\omega_0 - \gamma_0)/2|}
\]

(7)

We first consider the case of no Purcell effect spontaneous emission enhancement (\(F = 0\)) for which we have \(\omega_{00} = \omega_0\) and \(\gamma_{00} = \gamma_0\) and \(f_{3dB} = f_0\). Assuming further that the remaining damping term is relatively small, then the modulation bandwidth is well approximated by the resonance frequency which, via the photon lifetime, is inversely proportional to the group refractive index as indicated in (3) above. The latter is typically significantly enhanced in metal-clad nanolasers and hence, as noted by Ning [1], a trade-off exists between the enhanced net gain in nanolasers and the modulation bandwidth. In the case of surface plasmon polariton (SPP) modes, the refractive index enhancement can be half an order of magnitude, giving a corresponding reduction in achievable bandwidths.

Cavity effects: Consideration is now given to the situation where Purcell effect enhanced spontaneous emission is achieved in a metal-clad nanolaser, such enhancement being attractive in order to effect threshold current reduction in these devices. From the standpoint of laser dynamics several features can arise. First, the enhanced spontaneous emission, coupled with laser threshold reduction, can lead to a reduction of the laser turn-on delay. In principle, if threshold-less lasing is achieved then the turn-on delay would be reduced to zero. However, when consideration is given to the modulation performance of such lasers then attention needs to be given to the stronger damping which will result from enhanced spontaneous emission. Such damping will give rise to a long tail in the switch-off dynamics of the laser and hence will compromise both analogue and digital direct current modulation of the laser.

One clear consequence of such damping is a broadening of the resonance from which the maximum modulation frequency of the laser is deduced. Based on these physical considerations, a simple analysis has been performed to determine how the laser maximum modulation frequency is impacted by the combination of cavity enhanced damping and enhanced refractive indices consequent to the use of metal-cladding.

To examine these effects we write

\[
\gamma = \gamma_0 + \Delta \gamma
\]

(8)

where \(\gamma_0\) is the damping factor for no Purcell enhancement, i.e. \(F = 0\) in (6) above.

\[
\Delta \gamma = \beta F / \gamma_0 (1 + C_0 N_0 / S_0)
\]

(9)

As a first illustration of the significant effect of the enhanced damping we work with

\[
f_{3dB} = 1/2 \pi \sqrt{|(\omega_0^2 - \gamma_0^2)|}
\]

(10)

Assuming \(\omega_0 = \omega_{00}\) we have

\[
f_{3dB} = 1/2 \pi \sqrt{|(\omega_0^2 - \gamma_0^2)|}
\]

(11)

\[
f_{3dB} \geq 1/2 \pi \sqrt{|(\omega_0^2 - \gamma_0^2)|}
\]

(12)

Now \(\Delta \gamma \propto F\) and \(f_0 \propto 1/n_2\), hence \(\Delta f \propto F n_2\).
The above makes clear then that Purcell enhancement of spontaneous emission and enhanced group refractive index due to metal cladding combine deleteriously to reduce the nanolaser modulation bandwidth. The practical issue is the actual impact of these effects and to estimate the strength of these effects we examine $\Delta f/f_0 = \gamma_0 \Delta f/(2 f_0^2)$. As a first estimate we may assume $\gamma_0 = 1/\tau_s$, $f_0^2 = \tau_s \tau_p$, and $C_0 N_0/S_0 \gg 1$. Then $\Delta f/f_0 \simeq (\tau_p/\tau_s) \beta F C_0 N_0/S_0$. It is recalled that $\tau_p \propto n_p$ and so to capture the enhanced modal refractive index in metal-clad nanolasers we write $\tau_p = \tau_p/n_p n_0$ where $n_0$ is the background refractive index. Typically $\tau_p/\tau_s \sim 10^{-3}$, $N_0/S \sim 10^4$, $C_0 = 0.1$. Then it is estimated that $\Delta f/f_0 \simeq n_p/n_0 \beta F$. Taking a conservative value of $\beta F = 1$, then quite a large deterioration of the modulation bandwidth could arise with a combination of the Purcell effect and plasma-enhanced refractive index.

**Modulation bandwidth:** It is underscored that the preceding analysis has been presented in the given form in order to display explicitly the impact of key design features of metal-clad nanolasers. It is appreciated that more exact expressions for the modulation bandwidth can be obtained by retaining further terms in the expression for $f_{3\text{dB}}$ given in equation (4) above. In that case numerical evaluation of the bandwidth would be required but is beyond the scope of the present Letter.

**Conclusion:** It has been shown explicitly that key features of metal-clad nanolasers serve to compromise achievable modulation bandwidth in these devices. Simple expressions for the change in modulation bandwidth show that Purcell enhanced spontaneous emission and SPP enhanced group refractive indices work together to reduce the device 3 dB modulation bandwidth. It is shown that there is potential for significant impact on practical device operation. It is hoped that these results may inform the design of metal-clad nanolasers.

**Acknowledgments:** This work was undertaken during the tenure of a Japanese Society for the Promotion of Science (JSPS) invitation fellowship at the Graduate School of Materials in the Nara Institute of Science and technology (NAIST). Grateful acknowledgement is made of the support from JSPS as well as hospitality received from H. Kawaguchi, K. Ikeda and S. Koh and other staff and students in the Ultrafast Photonics Group in the Graduate School of Materials at NAIST. Helpful discussions with H. Kawaguchi, K. Ikeda and S. Koh are acknowledged.

© The Institution of Engineering and Technology 2010
8 September 2010
doi: 10.1049/el.2010.2535
K.A. Shore (Nara Institute of Science and Technology, Graduate School of Materials Science, Ultrafast Photonics, Ikoma, Nara 630-0192, Japan)
K. A. Shore: Permanent address: Bangor University, School of Electronic Engineering, Bangor, LL57 1 UT, Wales, United Kingdom

**References**