RESEARCH REPORT FOR JSPS LONDON POSTDOCTORAL FELLOWSHIP FOR FOREIGN RESEARCHERS (SHORT-TERM)

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HOST INSTITUTION: Kyoto University, Graduate School of Human and Environmental Studies

HOST RESEARCHER: Professor Yoshinori Morimoto

TITLE OF RESEARCH IN JAPAN: Microlocal Methods for kinetic equations

FELLOWSHIP PERIOD: From July 1st, 2010 to September 30th, 2010

MICROLOCAL METHODS FOR KINETIC EQUATIONS

My research project undertaken at Kyoto University under the supervision of Prof. Morimoto was concerned with the study of kinetic equations. Kinetic theory is a large area of partial differential equations in mathematical analysis dealing with the modeling of gases, plasmas or any system made up of a large number of particles, by a distribution function. This theory aims at explaining the macroscopic properties of these systems, such as pressure, temperature or volume, by considering their molecular composition and the motion of their particles.

More specifically, this research project was concerned with the study of the hypoelliptic properties for a class of kinetic equations which are linear models for the Boltzmann equation without angular cutoff. This class of kinetic operators has recently been studied by Prof. Chen, Li and Xu (Wuhan University), who proved some specific hypoelliptic estimates for this class of operators. However, these estimates are not optimal. The non-optimality for the hypoelliptic properties of these linear models is then a major obstacle when trying to take advantage of the understanding of the regularizing properties of linearized operators in order to investigate the properties of the general nonlinear Boltzmann equation without angular cutoff, which is the equation of main interest from the point of view of mathematical physics. By using more advanced microlocal techniques, we managed to improve the results obtained by Chen, Li and Xu; and to prove hypoelliptic estimates with optimal loss of derivatives.

During the fellowship, I also started another joint work with Prof. Lerner (Université Paris 6, France), Prof. Morimoto (Kyoto University, Japan) and Prof. Xu (Wuhan University, China) in which we begin to investigate the microlocal structure of the linear Kac's operator without angular cutoff. In this second work, we aim at disclosing more intimately the pseudodifferential nature of this linearized collision operator and at deriving optimal coercive estimates. The obtention of such optimal coercive estimates would be a significant progress in kinetic theory and would allow potential major breakthroughs for the study of the general Boltzmann equation without angular cutoff.

My stay at Kyoto University has also been a fantastic opportunity to initiate both formal and informal scientific interchanges with the Japanese academic community. I had the great honor to be invited by Prof. Iwasaki (University of Hyogo) to present my works in Kobe. I also had the pleasure to give a seminar talk at Nagoya University on the invitation of Prof. Sugimoto; and at Kyoto University during a special work session organized by Prof. Morimoto with Prof. Ukai (Tokyo Institute of Technology) and Prof. Fujiie (Ritsumeikan University). Furthermore, I had the nice opportunity to attend the Mathematical Society of Japan, Autumn Meeting 2010, held at Nagoya University on September $22-25^{\text{th}}$, 2010; where I met many other members of the Japanese academic community working in the domain of partial differential equations.

Finally, I would like to thank very warmly the Japan Society for the Promotion of Science and JSPS London for their magnificent supports which have made possible this exceptional opportunity to spend those three months at Kyoto University. I also would like to express my deep gratefulness to Prof. Morimoto and Kyoto University for their very kind hospitality and the exceptional working surroundings. This stay will be a lifetime experience and has been fantastic both on a scientific point of view as well as on a cultural one.

Karel Pravda-Starov