JSPS Fellowship Short Report: Rob Morris

I visited the Department of Mathematics at Keio University, Tokyo, from December 2008 to April 2009. My host was Prof. Katsuhiro Ota, a combinatorial mathematician who works on (amongst other things) the properties of graphs with forbidden minors. These graphs are interesting because, by the Robertson-Seymour Theorem, for every surface S there exists a finite family of forbidden minors which characterize the graphs which may be embedded into S. For example, the planar graphs are exactly those graphs with no $K_{3,3}$-minor and no K_5-minor (Wagner's Theorem).

The starting point for our research was a theorem of Tutte, who proved in 1956 that every 4-connected planar graph is hamiltonian (has a cycle through all its vertices). Tutte's Theorem provokes an obvious question: How long a cycle must exist in a 3-connected planar graph? This was an open problem for almost forty years, until 2002, when Chen and Yu proved that the answer is $\Theta(n^\alpha)$, where $\alpha = \log_3 2$. Their result was soon afterwards extended to 3-connected graphs on any surface (of bounded orientable genus).

Together with Kenta Ozeki, a student of Prof Ota, we decided to attack the following generalization of this problem: How long a cycle must exist in any 3-connected $K_{3,t}$-minor-free graph? Since every surface is $K_{3,t}$-minor-free for some $t \in \mathbb{N}$, the answer must be at most n^α, and Seymour and Thomas conjectured that it is at least n^ε for some $\varepsilon > 0$. We conjectured that the answer is in fact $\Theta(n^\alpha)$, and moreover formulated a much more general conjecture, which (if true) would be tight up to an absolute constant (independent of t). Although we have as yet been unable to prove our conjecture, we did succeed in proving it in the case which we expect to be extremal, and we are continuing our joint research on the problem by email.

Advice about living in Japan? First, remember that Japan is very different from the west! This can be both wonderful and frustrating, especially since Japanese people can be quite shy about explaining some of the cultural differences. Almost everyone is very friendly though, so don't be shy about going and saying hello to strangers! Second, don't expect people (outside the department) to speak English. Try to learn at least a hundred words of Japanese before you arrive, and your experience will be much better. Third, the food is amazing - experiment a lot, and get used to ordering and not knowing what you're going to get! Fourth, the winter is VERY cold, but the spring is wonderful - try to be there in April if you can. Finally, get out there and explore - the best things about Japan are hidden beneath the surface!

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