RESEARCH REPORT FOR JSPS INVITATION FELLOWSHIP (SHORT-TERM)

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Host: Professor Hiroshi Tamaru, Department of Mathematics, Hiroshima University
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Period of Fellowship: 1-31 August 2013

1 August: Arrival at Hiroshima University. My accommodation was in the International House of Hiroshima University (http://www.hiroshima-u.ac.jp/en/top/research_institutes/ihouse/). The location was convenient. It took me five minutes by bicycle from the International House to the Department, and ten minutes by bicycle to a large supermarket. Internet access was provided in the common room of the International House.

2 August: I participated in the International Half-Day Workshop on Differential Geometry at Hiroshima University (http://www.math.sci.hiroshima-u.ac.jp/~tamaru/sympo/13oneday.html

3 August: Discussion with Professor Sadahiro Maeda (Saga University) about ruled real hypersurfaces in complex hyperbolic spaces.

8-10 August: I participated in the *International Workshop on Special Geometry and Minimal Submanifolds* (http://www.math.tohoku.ac.jp/meetings/sgms/index-e.html) at Tohoku University. During the workshop I gave a one-hour lecture on "Polar actions on symmetric spaces".

11 August: I gave two one-hour lectures on "Cohomogeneity one actions on symmetric spaces" at the *Akihabara Differential Geometry Seminar* (http://tmugs.math.se.tmu.ac.jp/g-akihabara/130811.html).

12 August: Discussion with Professor Yoshiaki Maeda (Keio University) about joint UK-Japan activities for 2014 and 2015.

Other dates: Discussions with Professor Hiroshi Tamaru (Hiroshima University). This was the main purpose of my visit to Japan. Here is a brief research report:
An isometric action of a connected closed Lie group on a connected complete Riemannian manifold is said to be polar if the action admits a section. A section is a connected closed submanifold that intersects every orbit of the action orthogonally. Such a section is necessarily a totally geodesic submanifold. If the section is flat then the action is said to be hyperpolar. The general context of our discussions was the classification problem of polar actions on symmetric spaces of non-compact type. Some classifications are known for special cases, including cohomogeneity one actions, and hyperpolar actions without singular orbits (so-called homogeneous hyperpolar foliations). However, only few examples of polar actions that are not hyperpolar are known.

We investigated the case of polar actions with no singular orbits, that is, homogeneous polar foliations, under the additional assumption that one of the orbits is a totally geodesic submanifold. Such an orbit must then be a reflective submanifold. Algebraically this leads to two commuting involutions on the Lie algebra of the isometry group of the symmetric space. The underlying algebraic structure turns out to be very useful. We were able to reformulate the classification problem in algebraic terms and to solve it. The surprising result states that homogeneous polar foliations with a totally geodesic orbit on irreducible symmetric spaces of non-compact type exist only on real hyperbolic spaces, and all of them have a simple description up to orbit equivalence.
Our plan for the near future is to continue our research in this direction by dropping the assumption that there is no singular orbit. More precisely, the problem is to classify (up to orbit equivalence) all polar actions on irreducible symmetric spaces of non-compact type with a totally geodesic orbit. It is known that such actions exist on all symmetric spaces of non-compact type and rank one. For higher rank and the special case of cohomogeneity one we classified such actions in previous work, but for higher cohomogeneity our first investigations indicate that there may not exist such actions on symmetric spaces of higher rank unless they are hyperpolar and are orbit equivalent to known actions.

31 August: Departure from Hiroshima University. I had a very pleasant stay at Hiroshima University and the International House of the University and would like to emphasize that the environment provided by Hiroshima University for carrying out the fellowship program was excellent.

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